THE DESIGN OF A HYPERCARD STACK ON INTRODUCTORY CALCULUS

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INTRODUCTION

The purpose of this paper is to review several principles used to design a Hypercard stack for the learning of introductory calculus and the research questions related to the stack. The review, development and generation of research questions, being important phases of any research project, must be dealt with carefully to ensure that the research will produce significant and valid findings. As this project is still being developed, no empirical data will be reported here; feedback from the audience is most welcome to enhance the quality of this project.

THE PROBLEMS

Calculus is an important subject for further studies in the physical sciences, engineering and social sciences courses. Extensive research has identified a number of obstacles and misconceptions in the learning of introductory calculus. These include difficulties with concepts such as limits, continuity, tangents, differentiability and their notations (Dreyfus & Eisenberg, 1983; Orton, 1983; Tall, 1990; Vinner, 1987; Wong, 1984). Although many educators have actively promoted the use of graphing packages to teach function and calculus concepts, this enthusiasm needs to be further investigated because visual thinking demands a higher level of mental activity which students are often reluctant to engage in (Eisenberg & Dreyfus, 1990). A plausible but untested explanation is that students have not developed a repertoire of learning strategies to cope with various types of learning tasks beyond the ubiquitous practice strategy. This Hypercard stack is designed to address some of these problems. It is to be used in three different ways:

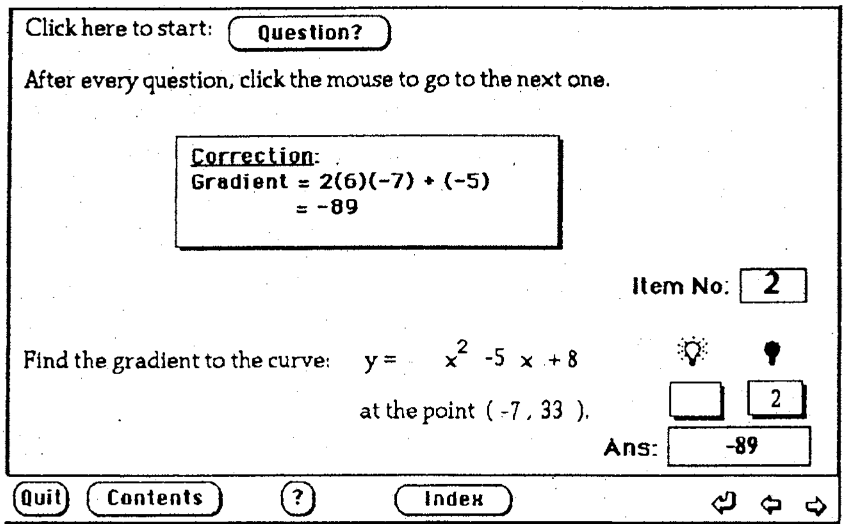
- as a self-paced learning package to supplement text materials;
- to encourage students' active participation in a group learning situation;
- to be used as a demonstration by the teacher who will model how mathematics is be learned and how learning tasks (especially investigations) are to be carried out.

THE PRESENTATION

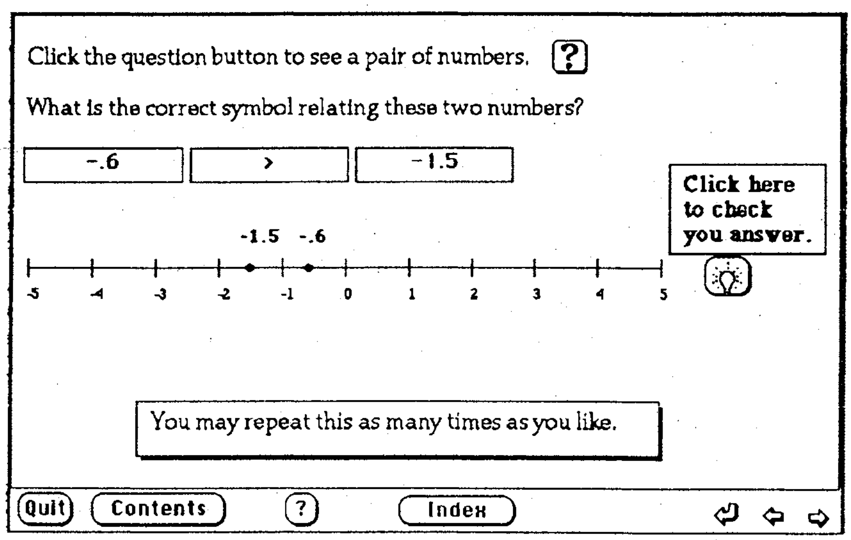
This paper is presented as a Hypercard stack and addresses four main aspects: feedback, different ways of doing things, graphing and user control in the Hypercard environment. The more important cards are shown below.

FEEDBACK

The *National Statement* states that, "Mathematical learning is likely to be enhanced by feedback" (p. 19). In traditional CAI, the computer evaluates the student's response and shows whether it is right or wrong, accompanied by some motivation such as an interesting graphics or a sound effect. This approach is illustrated by this card which shows a bright bulb for a correct answer and dark one for a wrong answer. A tally of the number of correct and wrong answers is also kept.

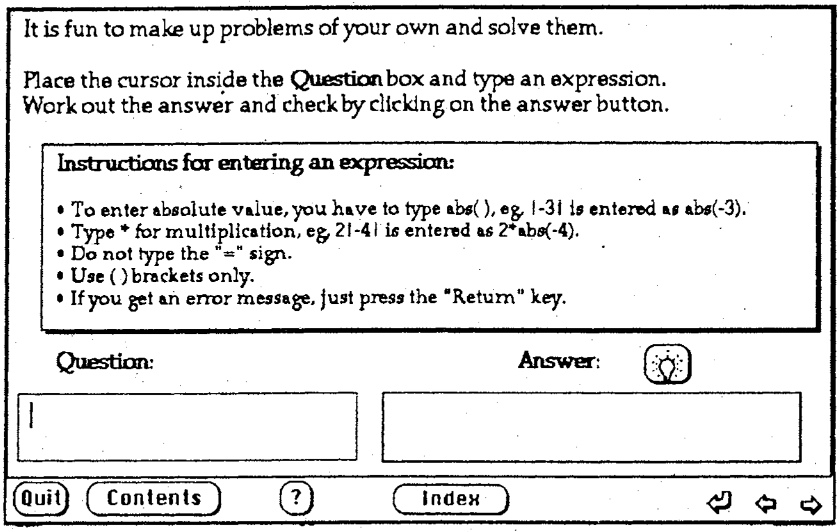


Another approach is that the computer outputs the correct answers without checking the students' responses. Additional information may be provided to enhance learning. This approach encourages the students to use the computer output to check their own answers. This is similar to checking answers at the back of the book and assumes that the students will take a greater responsibility of their own learning. The students also decide how much practice they need to do to master a particular skill. This approach is illustrated by this card.



The problem is to identify the correct symbol relating two numbers. When the student clicks on the answer button, the correct symbol is shown and a number line is given to consolidate the meaning of the symbol.

The third approach is to encourage students to make up their own questions and to use the computer to find the answers to these questions. In this card, the student make up computational questions involving different functions and operations. If this is done carefully the student can discover many properties about the functions and the precedence of various operations.



Given these different types of feedback, the research questions are to determine:

- students' preference for these types of feedback;
- the effects of feedback on learning outcomes and beliefs about learning;
- the types of questions made up by students and what they say about their mathematical knowledge.

DIFFERENT WAYS OF DOING THINGS

Mathematics educators emphasise that students should know different ways of solving the same problem. This will encounter a common belief that mathematics problem always has only one correct answer. This approach might not be feasible in actual teaching if the teacher feels the pressure to cover a lot of content. Research is also limited in determining how knowing different methods may affect problem solving behaviours and outcomes. The following cards show how several methods are used to solve the same absolute value equation. Exposing students to a combination of algebraic and graphical methods will enhance their ability to solve these equations in a flexible way and encourage them to represent mathematical knowledge in different forms.

Method 1:

Solve $|2x - 1| = 3$

Method 1 (Definition)

Treat $(2x - 1)$ as a unit as if you were solving $|x| = 3$.

So,

$$2x - 1 = -3 \quad \text{or} \quad 2x - 1 = 3$$

(Fill in the intermediate steps yourself!)

$$x = -1 \quad \text{or} \quad x = 2$$

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Method 2:

Solve $|2x - 1| = 3$

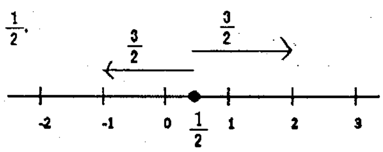
Method 2 (Number Line)

Change the equation to:

$|2x - 1| = |2(x - \frac{1}{2})| = 2|x - \frac{1}{2}| = 3$

so $|x - \frac{1}{2}| = \frac{3}{2}$

Find the two points which are $\frac{3}{2}$ from $\frac{1}{2}$.



Answers: -1, 2

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Method 3:

Solve $|2x - 1| = 3$

Method 3 (Square)

Square both sides: $(2x - 1)^2 = 9$ (Why?)

Expand and simplify:

$x^2 - x - 2 = 0$

$(x + 1)(x - 2) = 0$

$x = -1$ or 2

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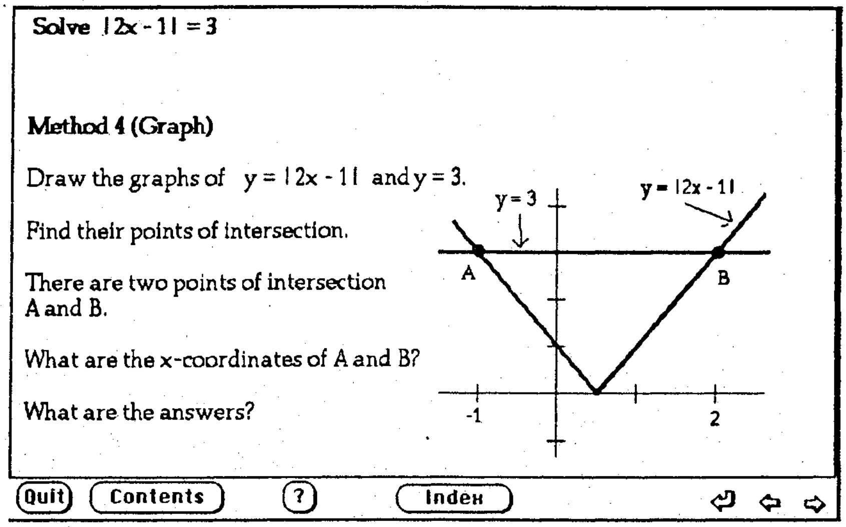
Method 4:

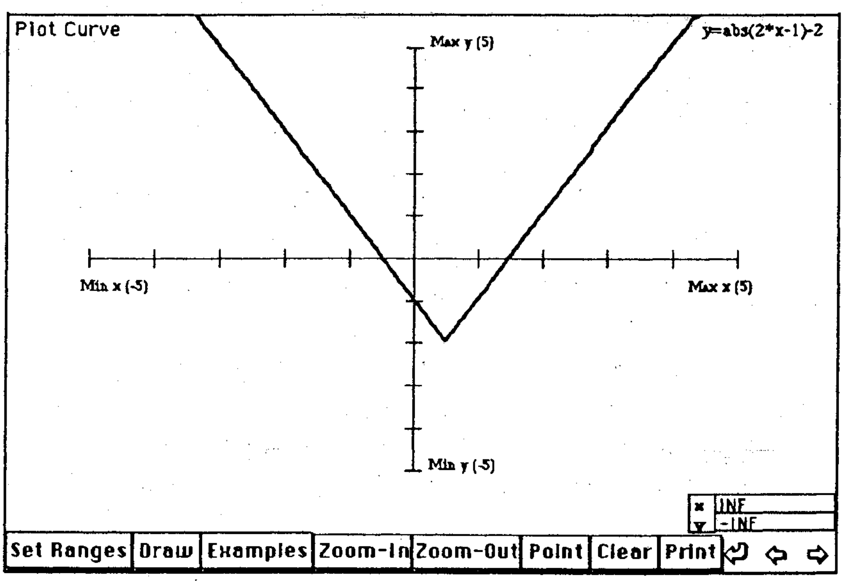


The research question is to determine:

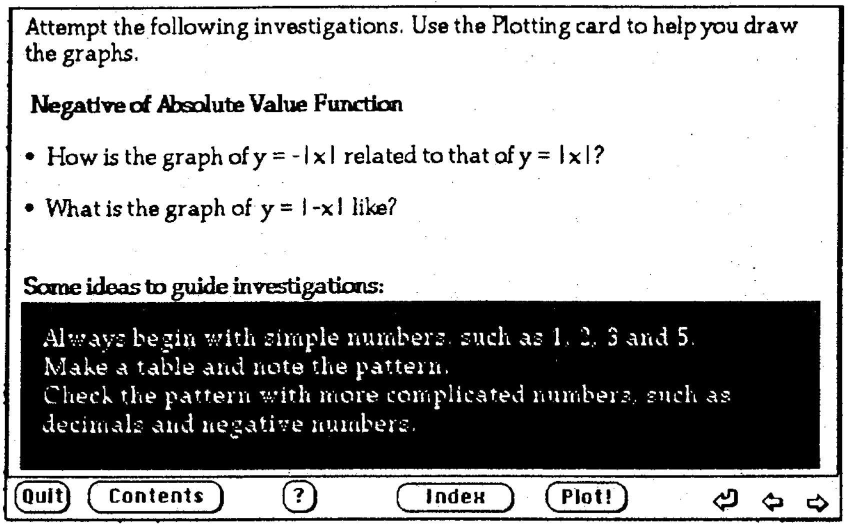
- students' preference for these approaches of solving the same problem; in particular, the preference for an analytical, visual, or a versatile mode of processing;
- the effects of multiple methods on learning outcomes and beliefs about learning.

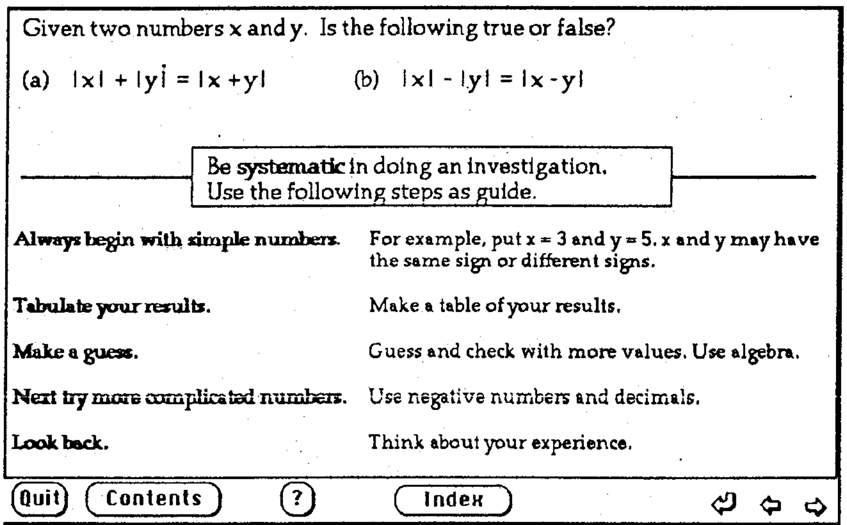
GRAPHING

There is considerable interest on visualisation in learning mathematics, in particular with the use of graphing software and graphic calculator. However, it is not realistic to expect students to be able to use these tools without adequate guidance. Since these graphing tools are for applications rather than for teaching, the guidance to use them as instructional tool must be provided for by the teachers, for examples in the form of worksheets. This stack attempts to cover both objectives by providing a plotting card together with ideas for investigations. The plotting card has the usual features of zoom-in and zoom-out, though it is a bit slow due to the speed of Hypercard processing.



Two examples of different guidelines for graphing are shown below.





The research question is to determine:

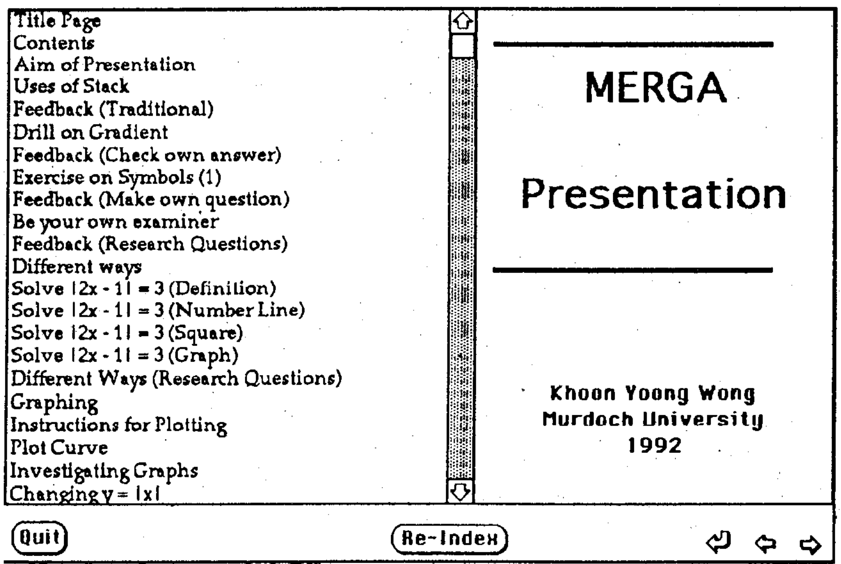
- whether students find these guidelines useful for doing investigations or not;
- the ways in which they use the plotting card as a tool to do their investigations;
- the effects of graphing on learning outcomes and beliefs about learning.

USER CONTROL

In computer-based instruction, it is often claimed that user control can improve learning although "[A]sking students to make too many decisions may actually interfere with the flow learning" (Steinberg, 1991, p.112). This is likely to affect low ability students or those who have poor study habits. However, in the Hypercard environment, "The user, not the computer, initiates and controls all actions" (*Hypercard Stack Design Guidelines*, 1989,

p. 180). The following features provide a consistent and user-friendly way of navigating through the stack:

- a set of clearly defined buttons placed at consistent locations;
- a Contents card to give an overview of what the stack is about;
- an Index card (see below) so that the user can access individual cards as needed;
- a GLOSSARY stack that allows the user to look up more information.



The research question is to determine:

- what kind of user control is preferred by students of different abilities;
- the effects of control on learning outcomes and beliefs about learning.

In order to obtain information on the ways the user moves through the stack, the study will make use of the ScreenRecorder software to record the Macintosh screen session in real time. This record can be played back to determine the learning path taken by the user to gain a better understanding of the learning process. However this method will collect an enormous amount of data and the most optimal way to analyse the data will be addressed at a later stage.

CONCLUSION

High expectations have been placed on using educational technology to facilitate teaching and learning. When the design of computer-based package is based on appropriate learning theories and design principles, there is great promise of realising these expectations. However, careful research is required to validate these possibilities. Hopefully some fruitful results will be obtained from this project in the near future.

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